



Yang, R., Yang, C., Chen, M., & Na, J. (2016). Robust control for robot manipulators with time-varying uncertainty based on bounded observer in discrete time. In *2016 22nd International Conference on Automation and Computing (ICAC 2016): Proceedings of a meeting held 7-8 September 2016, Colchester, United Kingdom* (pp. 395-400). [7604950] Institute of Electrical and Electronics Engineers (IEEE). <https://doi.org/10.1109/IConAC.2016.7604950>

Peer reviewed version

Link to published version (if available):  
[10.1109/IConAC.2016.7604950](https://doi.org/10.1109/IConAC.2016.7604950)

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# Robust Control for Robot Manipulators with Time-varying Uncertainty Based on Bounded Observer in Discrete Time

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**Abstract**—In this paper, we have developed a disturbance observer (DOB) based on robust control method for a class of nonlinear robot manipulators with time-varying uncertainty. To facilitate digital implementation of the controller, the robot system is formulated in discrete time. The DOB controller is design to compensate for uncertainty and disturbance by bounding both all states and observed uncertain function in a control region. The robust stability of closed-loop robot system can be well guaranteed by applying Schur complement theory and Lyapunov analysis, such that parameters of the DOB controller are derived using linear matrix inequalities (LMIs) theory. Simulation studies have been performed to test and verify the proposed control scheme, which results in supreme robust control and satisfied trajectory tracking performance for robot manipulators with time-varying uncertainty.

## I. INTRODUCTION

With advances of the technologies, nonlinear multiple-input multiple-output (MIMO) robot manipulators have been widely used in our modern life and industry. In practice, most robot manipulators are usually subject to unmodelled dynamics and various uncertainties [2]–[4], and many research works mainly focus on controller design to achieve satisfied control performance. In the recent decades, many control schemes are mainly designed via DOB with saturation states and/or external disturbances. For example, a robust adaptive controller is designed based on input saturation and observed external disturbance for uncertain nonlinear system in [20]. In [7], a state feedback controller is presented with input saturation and disturbance. In [8], a decentralized adaptive robust controller is proposed to realize trajectory tracking of robot manipulators. DOB methods are introduced to compensate for disturbances in these studies, and the technologies have increasingly matured and been used for robot manipulators in practice. However, for nonlinear, time-varying, uncertain robot manipulators, few results are reported and they are mainly designed in continuous time domain. For example, in [17],

robotic dynamic model is divided into known-model terms and un-modeled dynamics to compensate for uncertainty, and the controller is designed via a known system term, a feedback term and an adaptive term. In [9], neural network control method is introduced, which is comprised by a self-organizing neural-fuzzy network identifier, an uncertainty observer and a supervisory controller, furthermore, a sliding-mode control and an adaptive bound-estimation scheme are introduced to achieve satisfied control performances. In [16], a fuzzy adaptive output feedback control is proposed based on any observer to compensate for nonlinear uncertain system, but control system is a class of single-input-single-output (SISO). These approaches are able to guarantee stability of closed-loop robot systems and obtain satisfied control performances. Moreover, the digital implementation of robot controller is becoming increasingly popular and powerful, recent research works for robot manipulators with uncertainty gradually focus on discrete-time control.

Discrete-time robot manipulator models and discrete-time control methods compensating for nonlinear uncertainty are used in [5], [6], and these on-line control approaches are convenient to implement using discrete-time robot models. In [15], a discrete-time adaptive controller is designed for robot manipulators with unknown fixed or time-varying uncertain external payload to obtain a high quality trajectory tracking performance. These approaches perform well to guarantee robust, and many research results mainly focus on stability of nonlinear robot systems in discrete time. However, these schemes often assume that the uncertainty of robot manipulator is bounded in a fixed range, which limits their applications in practice.

In this paper, we develop a novel discrete-time DOB based on all system states and uncertain estimation of robot manipulators to compensate for these effect. Stability of the closed-loop system is able to be well guaranteed, and supreme trajectory tracking control performance is able to be achieved. Throughout the paper, the following notations are employed.

- $\|\cdot\|$  denotes the Euclidean norm of a vector or an induced

This work was supported in part by Engineering and Physical Sciences Research Council (EPSRC) under Grant EP/L026856/2, and Marie Curie Intra-European Fellowships Project AECE under Grant FP7-PEOPLE-2013-IEF-62553174.

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norm of matrix.

- $[\ ]^T$  represents the transpose of a vector or a matrix.
- $[\ ]^{-1}$  represents the inverse of a reversible matrix.
- $[\ ]^+$  represents the pseudo inverse of a singular matrix or a non-square matrix.
- $\mathbf{0}_m$  stands for a  $m$ -dimension zero vector.
- $\mathbf{I}_{n \times n}$  stands for a  $n$ -dimension unit matrix.
- $\mathbf{0}_{a \times b}$  stands for a  $a \times b$ -dimension zero matrix.

## II. PROBLEM FORMULATION

### A. Robot Manipulator Model

In the paper, the whole system studied includes an  $n$ -degrees of freedom (DOF) rigid class of robot manipulators, which are governed by the following dynamic model described in continuous-time:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau - \tau_d \quad (1)$$

where  $q \in \mathbb{R}^n$  is the joint angle,  $\dot{q} \in \mathbb{R}^n$  is the joint angle velocity,  $\ddot{q} \in \mathbb{R}^n$  denotes the joint angle acceleration,  $M(q) \in \mathbb{R}^{n \times n}$  is the symmetric positive definite inertia matrix,  $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$  is the Coriolis-Centrifugal torque matrix,  $G(q) \in \mathbb{R}^n$  denotes the gravity torque vector,  $\tau \in \mathbb{R}^n$  is the vector of control input torque,  $\tau_d(k) \in \mathbb{R}^n$  is the external disturbance torque vector.

According to [2], the system model (1) has the following properties hold:

*Property 1:* The symmetric and positive definite inertia matrix  $M(q)$  is uniformly bounded. There are constants  $m_1 > 0, m_2 > 0$ , and the matrix  $M(q)$  satisfies the following inequality

$$m_1 \leq \|M(q)\| \leq m_2 \quad (2)$$

*Property 2:* The matrix  $C(q, \dot{q})$  and the vector  $G(q)$  are bounded by  $\|C(q, \dot{q})\| \leq \rho_c \|\dot{q}\|$ , and  $\|G(q)\| \leq \rho_g$ , respectively, where  $\rho_c$  and  $\rho_g$  are positive constants.

### B. Robotic Discretization

The dynamic model represents a rigid class of robot manipulator in (1). Let us define  $\bar{q} = [q^T, \dot{q}^T]^T \in \mathbb{R}^{2n}$  and then the corresponding dynamics can be written as [15]

$$\dot{\bar{q}} = \Phi(q, \dot{q})\bar{q} + \Gamma(q) (\tau + \tau_d - G(q)) \quad (3)$$

with

$$\Phi(q, \dot{q}) = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{I}_{n \times n} \\ \mathbf{0}_{n \times n} & -M^{-1}(q)C(q, \dot{q}) \end{bmatrix} \in \mathbb{R}^{2n \times 2n}$$

$$\Gamma(q) = \begin{bmatrix} \mathbf{0}_{n \times n} \\ M^{-1}(q) \end{bmatrix} \in \mathbb{R}^{2n \times n}$$

The state equation can be discretized using the discretization theory with a small sampling time  $T$ , the sampled joint angle is described as  $q(k) = q(t_k)$ , the sampled joint angle velocity is described as  $\dot{q}(k) = \dot{q}(t_k)$ , the control torque is described as  $\tau(k) = \tau(t_k)$  and the external disturbance torque is written as  $\tau_d(k) = \tau_d(t_k)$  at the sampling time instant

$t_k = kT$ , respectively. Then, the equivalent dynamics form can be obtained as

$$\bar{q}(k+1) = L(k)\bar{q}(k) + H(k) (\tau(k) + \tau_d(k) - G(k)) \quad (4)$$

where  $\bar{q}(k) = [q^T(k), \dot{q}^T(k)]^T \in \mathbb{R}^{2n}$ , and  $G(k) = G(q(k))$  is the discretized gravity torque vector.  $L(k) \in \mathbb{R}^{2n \times 2n}$  and  $H(k) \in \mathbb{R}^{2n \times n}$  are counter-part discrete-time matrices, which correspond to the continuous-time matrices  $\Phi(q, \dot{q})$  and  $\Gamma(q)$  in (3), and the matrices  $L(k), H(k)$  can be calculated as following

$$L(k) = e^{\Phi(q(k), \dot{q}(k))T}, H(k) = \int_{(k-1)T}^{kT} e^{\Phi(q, \dot{q})t} \Gamma(q) dt \quad (5)$$

It is noted that we only can get the sampled values of  $q(k)$  and  $\dot{q}(k)$  at sampling time  $t_k = kT$ , and the matrix  $\Phi(k) = \Phi(q(k), \dot{q}(k))$  is determined at each sampling time. Consider uncertainty and unknown external disturbance of robot manipulators in (4) and (5), the counter-part matrices  $L(k), H(k)$  and gravity torque vector  $G(k)$  based on robot manipulator dynamics in discrete time can be constructed as

$$\begin{aligned} L(k) &= L_0 + \Delta L(k) \\ H(k) &= H_0 + \Delta H(k) \\ G(k) &= G_0 + \Delta G(k) \end{aligned} \quad (6)$$

where  $L_0 \in \mathbb{R}^{2n \times 2n}, H_0 \in \mathbb{R}^{2n \times n}, G_0 \in \mathbb{R}^n$  are known dynamics matrices, and  $\Delta L(k) \in \mathbb{R}^{2n \times 2n}, \Delta H(k) \in \mathbb{R}^{2n \times n}, \Delta G(k) \in \mathbb{R}^n$  are unknown terms of system dynamic parameters  $L(k), H(k), G(k)$ , respectively.

The assumption is employed as follows

*Assumption 1:* Assume the known matrix  $H_0$  has linearly independent columns, and thus the column rank of matrix  $H_0$  satisfies  $\text{rank}(H_0) = n$ , then, its left inverse  $H_0^+ \in \mathbb{R}^{n \times 2n}$  exists, and we have  $H_0^+ H_0 = \mathbf{I}_{n \times n}$ .

The aim of this paper is to obtain satisfied trajectory tracking performance under uncertain effect for robot manipulators, that is  $\lim_{k \rightarrow \infty} q(k) = q_d(k)$ ,  $q_d(k) \in \mathbb{R}^n$  is a desired ideal trajectory. Define a trajectory tracking error vector as

$$\xi(k) = \bar{q}(k) - \bar{q}_d(k) \quad (7)$$

where  $\xi(k) \in \mathbb{R}^{2n}$ , and  $\bar{q}_d(k) = [q_d^T(k), \dot{q}_d^T(k)]^T \in \mathbb{R}^{2n}$ . Substitute (7) and (6) into (4), we have

$$\begin{aligned} \xi(k+1) &= (L_0 + \Delta L(k))\xi(k) + (H_0 + \Delta H(k))\tau(k) \\ &\quad - (H_0 + \Delta H(k))(G_0 + \Delta G(k)) \\ &\quad + (H_0 + \Delta H(k))\tau_d(k) \\ &\quad + (L_0 + \Delta L(k))\bar{q}_d(k) - \bar{q}_d(k+1) \end{aligned} \quad (8)$$

Compounding these uncertain terms and external disturbance torque  $\tau_d(k)$  in (8), we define an unknown nonlinear function  $F(k) \in \mathbb{R}^{2n}$  as

$$\begin{aligned} F(k) &= F_0 f(k) \\ &= (H_0 + \Delta H(k))\tau_d(k) + \Delta H(k)\tau(k) \\ &\quad - \Delta H(k)G_0 - (H_0 + \Delta H(k))\Delta G(k) \\ &\quad + \Delta L(k)\bar{q}_d(k) \end{aligned} \quad (9)$$

where  $F_0 \in \mathbb{R}^{2n \times n}$  is a weight matrix, and  $f(k) = f(q(k), \dot{q}(k), q_d(k), \dot{q}_d(k), \tau(k), k) \in \mathbb{R}^n$  is also an unknown function corresponding with  $F(k)$  and is assumed as bounded. According to Assumption 1, we further define  $H_0 r(k) = L_0 \bar{q}_d(k) - \bar{q}_d(k+1)$  with an auxiliary vector  $r(k) = H_0^+(L_0 \bar{q}_d(k) - \bar{q}_d(k+1)) \in \mathbb{R}^n$ . Then, a standard actual discrete-time state form of the system (8) can be written as

$$\begin{aligned} \xi(k+1) &= (L_0 + \Delta L(k))\xi(k) + H_0 \tau(k) \\ &\quad - H_0 G_0 + H_0 r(k) + F_0 f(k) \end{aligned} \quad (10)$$

The system uncertain and unknown term  $\Delta L(k)$  in (10) is assumed to be of the following form [10]

$$\Delta L(k) = M_A \Lambda(k) N_A \quad (11)$$

where  $M_A \in \mathbb{R}^{2n \times a}$  and  $N_A \in \mathbb{R}^{b \times 2n}$  are two known matrices, and  $\Lambda(k) \in \mathbb{R}^{a \times b}$  is an unknown time-varying matrix, which satisfies

$$\Lambda(k)^T \Lambda(k) \leq \mathbf{I}, \forall k. \quad (12)$$

Note that  $\Delta L(k)$  is admissible if both (11) and (12) are satisfied.

*Remark 1:* It is noted that the structure of uncertain term in (11) has been widely using for research of robust control, and has been applied to resolve robust observer problems for both discrete-time systems and continuous-time systems. In many practical systems, it can be viewed as an appropriate presentation [11].

### III. DESIGN DOB FOR ROBOT SYSTEM WITH UNCERTAINTY

In this section, the DOB with saturation is designed to compensate for the effect of  $f(k)$  with uncertainty, disturbance and discretization error for robot manipulators.

The unknown nonlinear function vector  $f(k)$  defined in (9) and used in (10) can further be assumed by the following exosystem:

$$\begin{aligned} w(k+1) &= W w(k) \\ f(k) &= U w(k) \end{aligned} \quad (13)$$

where  $w(k) \in \mathbb{R}^n$  is the observer design vector for  $f(k)$ , and  $W \in \mathbb{R}^{n \times n}$  and  $U \in \mathbb{R}^{n \times n}$  are auxiliary matrices.

To solve uncertain effect for robot control, the saturation control method is applied, and the following assumption is considered as

*Assumption 2:* Assume  $\text{sat}(\phi(k))$  is a saturated nonlinear function, which is defined as

$$\text{sat}(\phi(k)) = [\text{sat}(\phi_1(k)), \dots, \text{sat}(\phi_n(k))]^T, i = 1, \dots, n \quad (14)$$

In this paper, we design the following bounded controller as

$$\tau_s(k) = \text{sat}(K_1 \xi(k) + K_2 \hat{f}(k)) \quad (15)$$

where  $K_1 \in \mathbb{R}^{n \times 2n}$ ,  $K_2 \in \mathbb{R}^{n \times n}$  are gain matrices, and  $\hat{f}(k)$  is the estimation of  $f(k)$ .

Furthermore, the system control input term  $\tau(k)$  in (10) is designed as

$$\tau(k) = H_0^+(H_0 \tau_s(k) + H_0 G_0 - H_0 r(k)) \quad (16)$$

where  $H_0^+$  is the left inverse of  $H_0$  satisfying  $H_0^+ H_0 = \mathbf{I}_{n \times n}$  according to Assumption 1.

For designing the DOB with saturation, we use the following these Lemmas and Definition as

*Lemma 1:* [13] Assume that  $\mathcal{D} = \{D_1, D_2, \dots, D_{2^n}\}$  is the set of  $n \times n$  diagonal matrices with diagonal elements being either 1 or 0, if  $D_l \in \mathcal{D}$ , we obtain that  $D_l^- = \mathbf{I}_n - D_l$  with  $l = 1, 2, \dots, 2^n$ .

*Lemma 2:* [14] Assume that control input  $\tau_s(k) \in \mathbb{R}^n$  and  $v(k) = [v_1, \dots, v_n]^T \in \mathbb{R}^n$ , if  $|v_i| \leq \tau_{imax}$ , the saturated input  $\text{sat}(\tau_s(k))$  can be represented as

$$\text{sat}(\tau_s(k)) = \sum_{l=1}^{2^n} \beta_l (D_l \tau_s(k) + D_l^- v(k))$$

where  $i = 1, \dots, n$ ,  $0 < \beta_l < 1$ ,  $\sum_{l=1}^{2^n} \beta_l = 1$ .

*Definition 1:* [12] The control input  $\tau_s(k)$  can be saturated in a  $\tau_{imax}$  linear region, which is defined as

$$\wp(V_1, V_2) = (\xi(k), \hat{f}(k)) : \|V_{i,1} \xi(k) + V_{i,2} \hat{f}(k)\| \leq \tau_{imax} \quad (17)$$

where  $\wp(V_1, V_2) \in \mathbb{R}^{3n}$ ,  $V_1 = [V_{1,1}, \dots, V_{2n,1}]^T \in \mathbb{R}^{n \times 2n}$  with  $V_{i,1} \in \mathbb{R}^{1 \times 2n}$ ,  $V_2 = [V_{1,2}, \dots, V_{n,2}]^T \in \mathbb{R}^{n \times n}$  with  $V_{i,2} \in \mathbb{R}^{1 \times n}$ , and  $i = 1, 2, \dots, n$ .

According to Lemmas 1 and 2, and Definition 1, we can assume  $v_i = V_{i,1} \xi(k) + V_{i,2} \hat{f}(k)$ , which satisfies  $|v_i| \leq \tau_{imax}$ , then, the control input  $\tau(k)$  in (16) can be saturated in  $\tau_{imax}$  and be represented as

$$\begin{aligned} \tau_s(k) &= \text{sat}(K_1 \xi(k) + K_2 \hat{f}(k)) \\ &= \sum_{l=1}^{2^n} \beta_l D_l (K_1 \xi(k) + K_2 \hat{f}(k)) \\ &\quad + \sum_{l=1}^{2^n} \beta_l D_l^- (V_1 \xi(k) + V_2 \hat{f}(k)) \end{aligned} \quad (18)$$

In equation (18), the estimation value  $\hat{f}(k)$  can be obtained by designing the following observer

$$\begin{cases} \hat{w}(k) = g(k) - K_3 \xi(k) \\ g(k+1) = (W + K_3 F_0 U) \hat{w}(k) + K_3 (L_0 \xi(k) + H_0 \tau(k) - H_0 G_0 + H_0 r(k)) \end{cases} \quad (19)$$

where  $w(k)$  is defined in (13),  $g(k) \in \mathbb{R}^n$  is an auxiliary vector in the observer,  $K_3 \in \mathbb{R}^{n \times 2n}$  is design as feedback gain matrix.

Considering (13), (10) and (19), the estimation error  $\tilde{f}(k) = \hat{f}(k) - f(k)$  can be obtained as

$$\begin{aligned} \tilde{w}(k+1) &= \hat{w}(k+1) - w(k+1) \\ &= (W + K_3 F_0 U) \tilde{w}(k) - K_3 \Delta L(k) \xi(k) \end{aligned} \quad (20)$$

Substituting (18) and (16) into (10), the closed loop system is formulated by

$$\xi(k+1) = \sum_{l=1}^{2^n} \beta_l \{ (L_0 + \Delta L(k) + R)\xi(k) + \bar{L}\tilde{w}(k) + \bar{H}w(k) \} \quad (21)$$

where  $R = H_0(D_l K_1 + D_l^- V_1) \in \mathbb{R}^{2n \times 2n}$ ,  $\bar{L} = H_0(D_l K_2 + D_l^- V_2)U$  and  $\bar{H} = (H_0 D_l K_2 + H_0 D_l^- V_2 + F_0)U \in \mathbb{R}^{2n \times n}$ . The system (21) and the uncertain function error (20) can be combined and formulated as

$$\bar{\xi}(k+1) = \sum_{l=1}^{2^n} \beta_l \{ A_s(k)\bar{\xi}(k) + H_s w(k) \} \quad (22)$$

with

$$\begin{aligned} \bar{\xi}(k) &= \begin{bmatrix} \xi(k) \\ \tilde{w}(k) \end{bmatrix}, & H_s &= \begin{bmatrix} \bar{H} \\ \mathbf{0} \end{bmatrix} \\ A_s(k) &= \begin{bmatrix} L_0 + \Delta L(k) + R & \bar{L} \\ -K_3 \Delta L(k) & W + K_3 F_0 U \end{bmatrix} \end{aligned} \quad (23)$$

The observer (19) is designed to compensate for the unknown uncertain function  $f(k)$ . Given all initial values, the system state  $\xi(k+1)$  and unknown function estimation error  $\tilde{w}(k)$  can be computed and analysed based on (22).

Stability of robot system and control performance can be analyzed by the following proof.

#### IV. CONTROLLER REALIZATION AND STABLE ANALYSING

The design parameter matrices  $K_1, K_2, K_3, V_1, V_2$  of the observer are designed to guarantee the closed control system (22) asymptotically stable. These parameters are able to be obtained by applying the following Schur complement theorem and stability method as

*Lemma 3:* [19] Given the symmetric constant matrices  $S_{11}, S_{22}$  and constant matrix  $S_{12}$ , then,  $S_{22} < 0$  and  $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$  hold if and only if

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0 \quad (24)$$

*Lemma 4:* [18] Assume  $\mathcal{Y}, \mathcal{A}$  and  $\mathcal{B}$  are real matrices with approximate dimensions,  $\mathcal{Y}$  is a symmetric matrix, and  $X^T X \leq \mathbf{I}$ , then  $\mathcal{Y} + \mathcal{A}X\mathcal{B} + \mathcal{B}^T X^T \mathcal{A}^T < 0$  holds if and only if there exists a scalar  $\alpha > 0$  such that

$$\mathcal{Y} + \alpha \mathcal{A} \mathcal{A}^T + \alpha^{-1} \mathcal{B}^T \mathcal{B} < 0 \quad (25)$$

Definite the Lyapunov function as

$$V(k) = \bar{\xi}^T(k) P \bar{\xi}(k) \quad (26)$$

where  $P \in \mathbb{R}^{3n \times 3n}$  is a symmetric positive defined matrix, which can guarantee the closed system is stable. Then, we assume that the matrix  $P$  exists, and is defined as

$$P(k) = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} = \begin{bmatrix} Q_1^{-1} & 0 \\ 0 & P_2 \end{bmatrix} > 0 \quad (27)$$

with  $P_1 = Q_1^{-1} \in \mathbb{R}^{2n \times 2n} > 0$  and  $P_2 \in \mathbb{R}^{n \times n} > 0$ .

Then,  $\Delta V(k) = V(k+1) - V(k)$  is further analyzed that

$$\Delta V(k) \leq \max_{l \in [1, 2^n]} \begin{bmatrix} \bar{\xi}(k) \\ w(k) \end{bmatrix}^T S_1 \begin{bmatrix} \bar{\xi}(k) \\ w(k) \end{bmatrix} \quad (28)$$

where  $S_1$  is a matrix, which represents as

$$S_1 = \begin{bmatrix} A_s^T P A_s - P & A_s^T P H_s \\ H_s^T P A_s & H_s^T P H_s \end{bmatrix} \quad (29)$$

It is obvious that  $\Delta V < 0$  in (28) holds if  $S_1 < 0$ .

Apply the Schur complement theory in (29), a new matrix  $S_2 < 0$  can be obtained from matrix  $S_1 < 0$ , and there has  $S_2 < 0 \Leftrightarrow S_1 < 0$ , such that the matrix  $S_2$  can be derived as

$$S_2 = \begin{bmatrix} -P & * & * \\ \mathbf{0} & \mathbf{0} & * \\ A_s & H_s & -P^{-1} \end{bmatrix} < 0 \quad (30)$$

Thus, it is shown that  $\Delta V < 0$  holds if and only if  $S_2 < 0$  for positive symmetric defined matrix  $P$ .

Substituting (27), (22) and (11) into (30), we have

$$\begin{aligned} S_3 &= \begin{bmatrix} -P_1 & * & * & * & * \\ \mathbf{0} & -P_2 & * & * & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & * & * \\ L_0 + R & \bar{L} & \bar{H} & -P_1^{-1} & * \\ \mathbf{0} & Z & \mathbf{0} & \mathbf{0} & -P_2^{-1} \end{bmatrix} \\ &+ E \Lambda(k) \Upsilon + \Upsilon^T \Lambda^T(k) E^T < 0 \end{aligned} \quad (31)$$

where  $Z = W + K_3 F_0 U$ , and matrices  $E, \Upsilon$  are defined as

$$E = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ M_A \\ -K_3 M_A \end{bmatrix} \quad \Upsilon^T = \begin{bmatrix} N_A^T \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

Applying Lemmas 3 and 4, we see that the LIM (31) can be equivalently transformed as a new matrix  $S_4$ . It is shown that the  $S_3 < 0$  in (31) holds if and only if the following new matrix  $S_4 < 0$ , such that

$$S_4 = \begin{bmatrix} \pi_{11} & * & * \\ \pi_{21} & \pi_{22} & * \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} < 0 \quad (32)$$

with

$$\begin{aligned} \pi_{11} &= \begin{bmatrix} -P_1 & * & * \\ \mathbf{0} & -P_2 & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} & \pi_{21} &= \begin{bmatrix} L_0 + R & \bar{L} & \bar{H} \\ \mathbf{0} & Z & \mathbf{0} \end{bmatrix} \\ \pi_{22} &= \begin{bmatrix} -P_1^{-1} & * \\ \mathbf{0} & -P_2^{-1} \end{bmatrix} & \pi_{31} &= \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ N_A & \mathbf{0} & \mathbf{0} \end{bmatrix} \\ \pi_{32} &= \begin{bmatrix} M_A^T & -M_A^T K_3^T \\ \mathbf{0} & \mathbf{0} \end{bmatrix} & \pi_{33} &= \begin{bmatrix} -\alpha^{-1} \mathbf{I} & * \\ \mathbf{0} & -\alpha \mathbf{I} \end{bmatrix} \end{aligned}$$

where  $\alpha > 0$  is a given real constant.

Furthermore, we can define auxiliary matrix

$$\begin{aligned} \Omega_1 &= \text{diag} \{ P_1^{-1}, \mathbf{I}_n, \mathbf{I}_n, \mathbf{I}_{2n}, \mathbf{I}_n, \mathbf{I}_a, \mathbf{I}_b \} \\ \Omega_2 &= \text{diag} \{ \mathbf{I}_{2n}, \mathbf{I}_n, \mathbf{I}_n, \mathbf{I}_{2n}, P_2, \mathbf{I}_a, \mathbf{I}_b \} \end{aligned}$$

where subscript  $a$  and  $b$  describe column's dimension of matrix  $M_A$  and row's dimension of matrix  $N_A$ , respectively.

Thus, a new matrix  $S_5 = \Omega_2^T (\Omega_1^T S_4 \Omega_1) \Omega_2$  can be obtained as

$$S_5 = \begin{bmatrix} \Pi_{11} & * & * \\ \Pi_{21} & \Pi_{22} & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} \end{bmatrix} < 0 \quad (33)$$

with

$$\begin{aligned}\Pi_{11} &= \begin{bmatrix} -Q_1 & * & * \\ \mathbf{0} & -P_2 & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \\ \Pi_{21} &= \begin{bmatrix} O_1 & \bar{L} & \bar{H} \\ \mathbf{0} & O_2 & \mathbf{0} \end{bmatrix}, \quad \Pi_{22} = \begin{bmatrix} -Q_1 & * \\ \mathbf{0} & -P_2 \end{bmatrix} \\ \Pi_{31} &= \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ N_A Q_1 & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \Pi_{32} = \begin{bmatrix} M_A^T & -M_A^T X_3^T \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\ \Pi_{33} &= \pi_{33}\end{aligned}$$

where  $O_1 = L_0 Q_1 + H_0 D_l X_1 + H_0 D_l^- X_2$ ,  $O_2 = P_2 W + X_3 F_0 U$ .

It is shown that  $S_5 < 0 \Leftrightarrow S_4 < 0$ , such that we have  $\Delta V(k) < 0$  if and only if  $S_5 < 0$ , which implies that  $q(k) \rightarrow q_d(k)$  and  $\tilde{w}(k) \rightarrow 0$  as  $k \rightarrow \infty$ . Thus, the following Theorem can be described as

**Theorem 1:** For the given parameter  $\alpha > 0$ , giving auxiliary matrices  $U, W, M_A, N_A$ , if there exists symmetric positive-defined matrices  $P_1 = Q_1^{-1} > 0, P_2 > 0$ , and if matrices  $X_1, X_2, X_3, K_1, K_2, K_3, V_1, V_2$  satisfy  $S_5 < 0$ , then, the closed-loop system in (22) is asymptotically stable based on the DOB controller, the robot system with uncertainty has satisfied trajectory tracking performance under the following parameter designed as  $K_1 = X_1 Q_1^{-1}, V_1 = X_2 Q_1^{-1}, K_3 = P_2^{-1} X_3$ .

The proof is completed.

## V. SIMULATION STUDIES

To verify the trajectory tracking control performance of the above developed robust controller with DOB, a testing example, 2-DOF robot manipulator with uncertainty and nonlinear, external disturbance, is put forward in this section.

### A. Robot Manipulator Dynamics Model

The parameters of the robot manipulator are given as follows [15]: the mass are  $m_1 = m_2 = 1.0\text{kg}$ , the length are  $l_1 = l_2 = 0.2\text{m}$ , the inertia are  $I_1 = I_2 = 0.003\text{kgm}^2$ , the distance are  $l_{c1} = l_{c2} = 0.1\text{m}$ .

The dynamics of a robot manipulator with  $G(q) = [G_1, G_2]^T$  is given as

$$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, C(q, \dot{q}) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad (34)$$

where

$$M_{11} = m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos(q_2)) + I_1 + I_2$$

$$M_{12} = M_{21} = m_2 (l_1 l_{c2} \cos(q_2) + I_2)$$

$$M_{22} = m_2 l_{c2}^2 + I_2$$

$$C_{11} = -m_2 l_1 l_{c2} \sin(q_2) \dot{q}_2$$

$$C_{12} = -m_2 l_1 l_{c2} \sin(q_2) (\dot{q}_1 + \dot{q}_2)$$

$$C_{21} = m_2 l_1 l_{c2} \sin(q_2) \dot{q}_1, \quad C_{22} = 0$$

$$G_1 = m_1 l_{c1} g \cos(q_1) + m_2 g (l_{c2} \cos(q_1 + q_2) + l_1 \cos(q_1))$$

$$G_2 = m_2 l_{c2} g \cos(q_1 + q_2)$$

$$I_1 = \frac{1}{3} m_1 l_{c1}^2, \quad I_2 = \frac{1}{3} m_2 l_{c2}^2$$

Then, consider the robot manipulator with uncertainty and disturbance, the known dynamic parameters are assumed as:

$$\begin{aligned}L_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, H_0 = F_0 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & -0.3 \\ -0.1 & 1 \\ 1 & 0.1 \end{bmatrix} \\ G_0 &= \begin{bmatrix} 0.0001 \\ 0.0001 \end{bmatrix}, M_A = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{bmatrix}, N_A = [-0.1, 0.3, 0.4, 1]\end{aligned}$$

The matrices of exogenous system in (13) are chosen as:

$$U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, W = \begin{bmatrix} 0.001 & 0.0001 \\ -0.0002 & 0.0003 \end{bmatrix}$$

The initial state values of the robot system are assumed as  $\bar{q}(0) = [0, 0, 0, 0]^T$ ,  $\tau(0) = [0, 0]^T$ ,  $w(0) = [0, 0]^T$ ,  $g(0) = [0, 0]^T$ . The linear regions of control input  $\tau(k)$  are chosen as  $\tau_{1max} = 100$  and  $\tau_{2max} = 60$ . The constant  $\alpha$  is given as  $\alpha = 5$ .

Based on Theorem 1, the following parameters can be obtained by using LMIs theory as:

$$\begin{aligned}K_1 &= \begin{bmatrix} 50 & -6 & -5.5 & -0.5 \\ -5 & -20 & -2 & -1 \end{bmatrix} \\ K_2 &= \begin{bmatrix} -5.5 & -0.5 \\ -2 & -1 \end{bmatrix} \\ K_3 &= \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ -0.1 & 0.2 & 0.1 & 0.3 \end{bmatrix} \\ V_1 &= \begin{bmatrix} 0.0853 & -0.2924 & -0.0794 & -0.9348 \\ 0.0355 & -0.0120 & -0.8200 & -0.1431 \end{bmatrix} \\ V_2 &= \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}\end{aligned}$$

and the positive symmetric defined matrix  $Q_1, P_2$  are derived as:

$$\begin{aligned}Q_1 &= P_1^{-1} = 1.0e + 08 \times \\ &\begin{bmatrix} 1.4221 & 0.1276 & 0.2542 & 0.0023 \\ 0.1276 & 1.1777 & -0.4382 & -0.1653 \\ 0.2542 & -0.4382 & 1.3370 & -0.3779 \\ 0.0023 & -0.1653 & -0.3779 & 0.2010 \end{bmatrix} \\ P_2 &= 1.0e + 03 \times \begin{bmatrix} 3.3441 & -5.6150 \\ -5.5181 & 9.2712 \end{bmatrix}\end{aligned}$$

### B. Test Results

The external force torque  $\tau_d$  and the desired trajectory  $q_d$  are taken as below

$$\begin{aligned}\tau_d &= \begin{bmatrix} 0.05 \cos(0.01t) \cos(q_1) \\ 0.05 \cos(0.01t) \cos(q_2) \end{bmatrix} \\ q_d &= \begin{bmatrix} 1.5 + 0.5(\sin(0.3t) + \sin(0.2t)) \\ 1.5 + 0.5(\cos(0.4t) + \sin(0.3t)) \end{bmatrix}\end{aligned}$$

We construct the observer to compensate for uncertainty and disturbance of robotic manipulators by saturating system state  $\tilde{\xi}(k)$  and estimation of uncertain function  $\hat{f}(k)$ . The following simulation results are presented with the controller sampling

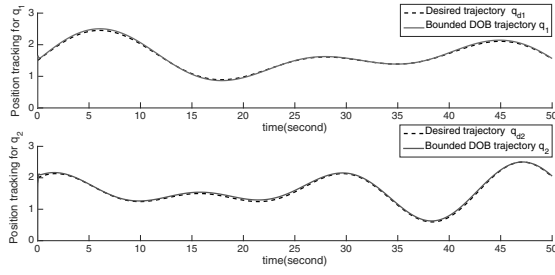


Fig. 1. Trajectory tracking curves  $q_1$  and  $q_2$

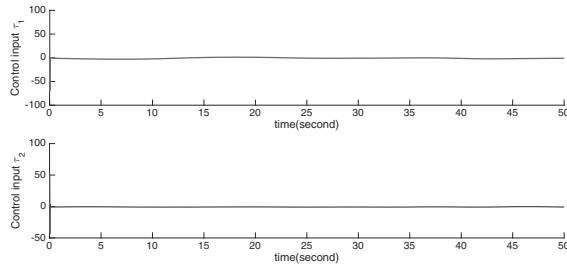


Fig. 2. Control input curves  $\tau_1$  and  $\tau_2$

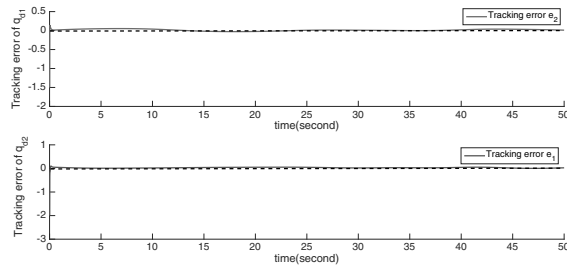


Fig. 3. Position tracking error curves

interval  $T = 0.01s$  in a very short period of time.

To show the effectiveness, using above design parameters  $K_1, K_2, K_3, V_1, V_2$ , the trajectory tracking control results of robot manipulator with uncertainty are shown in Figs.1-3. Fig.1 shows trajectory tracking curves of  $q_1$  joint position and  $q_2$  joint position, Fig.2 shows control input signals  $\tau_1$  and  $\tau_2$ , and Fig.3 shows position tracking error of  $q_1$  and  $q_2$  for desired trajectory  $q_{d1}$  and  $q_{d2}$ . Analyzing all above simulation results, the joints have an initial errors, which are away from the desired trajectories for less than  $0.02s$ , but the proposed control regulates the system trajectory quickly to achieve the desired trajectory and guarantees overall control stability. The whole control process is smooth, stable and accurate.

## VI. CONCLUSION

In this paper, a novel controller with DOB has been studied for discrete-time nonlinear MIMO robot manipulators with uncertainty and disturbance. The discrete-time DOB is proposed based on saturation for all system states and all uncertain signs including disturbance to compensate for these uncertain

influences, the method not only is able to guarantee the system is Lyapunov stability, but also is able to achieve the satisfied trajectory tracking performance.

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